SOLUTIONS CHAPTER 6 EXERCISES

1. Several reactor configurations are to be considered for reducing the influent substrate concentration from 100 mg/L to 20 mg/L at a design flow rate of 1 million gallons per day. Assume that substrate removal follows first-order kinetics and the first-order rate constant is 0.8 d\(^{-1}\). The following equation may be used for estimating the removal efficiency for completely-mixed reactors operating in series assuming first-order removal reactions.

\[
\frac{S_n}{S_o} = \left( \frac{1}{1 + k \theta} \right)^n
\]

Where \(S_o\) and \(S_n\) = Influent substrate concentration and effluent substrate concentration from the \(n^{th}\) completely-mixed reactor in series, mass/volume; \(n\) = Number of completely-mixed reactors in series; \(k\) = First-order removal rate coefficient, d\(^{-1}\), and \(\theta\) = Detention time in each of the completely-mixed reactors in series. Determine and compare the volume required for the following reactor configurations.

a. One continuous flow, ideal completely mixed reactor

From Table 6.3, use the following equation:

\[
\theta = \frac{(C_o/C_t - 1)}{k} = \frac{(100 \text{ mg/L}/20 \text{ mg/L} - 1)}{0.8 \text{ d}^{-1}} = 5 \text{ days}
\]

\[
V = \theta Q = 5 \text{ days} \left( \frac{1 \text{ MG}}{d} \right) = 5 \text{ MG or } 6.68 \times 10^5 \text{ ft}^3
\]

b. One continuous flow, ideal plug flow reactor

From Table 6.4, use the following equation:

\[
\theta = \frac{\ln(C_o) - \ln(C_t)}{k} = \frac{\ln(100 \text{ mg/L}) - \ln(20 \text{ mg/L})}{0.8 \text{ d}^{-1}} = 2.01 \text{ days}
\]

\[
V = \theta Q = 2.01 \text{ days} \left( \frac{1 \text{ MG}}{d} \right) = 2.01 \text{ MG or } 2.69 \times 10^5 \text{ ft}^3
\]

c. Two continuous flow, ideal completely mixed reactors in series

\[
\frac{S_n}{S_o} = \left( \frac{1}{1 + k \theta} \right)^n \quad \frac{20 \text{ mg/L}}{100 \text{ mg/L}} = \left( \frac{1}{1 + 0.8 \text{ d}^{-1} \theta} \right)^2
\]
\[ \left( \frac{20}{100} \right)^{\frac{1}{2}} = \frac{1}{1 + 0.8 \theta} \]

\[ 0.4472(1 + 0.8 \theta) = 1 \quad \theta = 1.55 \text{ days} \]

\[ V = 0 Q = 1.55 \text{ days} \left( \frac{1 \text{ MG}}{\text{d}} \right) = 1.55 \text{ MG or } 2.07 \times 10^5 \text{ ft}^3 \]

d. Four continuous flow, ideal completely mixed reactors in series

\[ \frac{S_n}{S_o} = \left( \frac{1}{1 + k \theta} \right)^n \quad \frac{20 \text{ mg/L}}{100 \text{ mg/L}} = \left( \frac{1}{1 + 0.8 \text{ d}^{-1} \theta} \right)^4 \]

\[ \left( \frac{20}{100} \right)^{\frac{1}{4}} = \frac{1}{1 + 0.8 \theta} \]

\[ 0.6687(1 + 0.8 \theta) = 1 \quad \theta = 0.619 \text{ days} \]

\[ V = 0 Q = 0.619 \text{ days} \left( \frac{1 \text{ MG}}{\text{d}} \right) = 0.619 \text{ MG or } 8.28 \times 10^4 \text{ ft}^3 \]

2. During a chemical reaction, the concentration of Species A was measured as a function of time. The observed concentration at various time intervals is presented below. Determine the reaction order and rate constant, k. Is Species A being removed or produced?

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Concentration of A (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
</tr>
</tbody>
</table>

A plot of concentration of A versus time on arithmetic paper produces a straight line indicating a zero order reaction. The slope of the line is equal to \( k = 2 \) mg/L·min. Species A is being removed. See plot on next page.
3. The concentration of Species C was measured as a function of time during a chemical reaction. The observed concentration of Species C at various time intervals is presented below. Determine the reaction order and rate constant, k. Is Species C being removed or produced?

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Concentration of C (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1.0</td>
<td>80</td>
</tr>
<tr>
<td>3.5</td>
<td>50</td>
</tr>
<tr>
<td>6.5</td>
<td>26</td>
</tr>
<tr>
<td>11.0</td>
<td>10</td>
</tr>
</tbody>
</table>

A plot of the natural log of C versus time produces a straight line on arithmetic paper indicating a first order reaction. The slope of the line is equal to \( k = 0.21 \text{hr}^{-1} \). Species C is being removed. See plot below.
4. The concentration of Species D was measured as a function of time during a chemical reaction. The observed concentration of Species D at various time intervals is presented below. Determine the reaction order and rate constant, k. Is Species D being removed or produced?

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Concentration of D (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>1.0</td>
<td>142</td>
</tr>
<tr>
<td>2.0</td>
<td>111</td>
</tr>
<tr>
<td>3.0</td>
<td>90</td>
</tr>
<tr>
<td>4.0</td>
<td>77</td>
</tr>
<tr>
<td>5.0</td>
<td>67</td>
</tr>
</tbody>
</table>

A plot of the reciprocal of D versus time produces a straight line on arithmetic paper indicating a second order reaction. The slope of the line yields $k = 0.002 \text{ L/mg·h}$. Species D is being removed. See plot below.

5. During a chemical reaction, the concentration of Species B was measured as a function of time. The observed concentration of Species B at various time intervals is presented below. Determine the reaction order and rate constant, k. Is Species B being removed or produced?

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Concentration of B (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>5.0</td>
<td>125</td>
</tr>
<tr>
<td>10.0</td>
<td>150</td>
</tr>
<tr>
<td>15.0</td>
<td>175</td>
</tr>
<tr>
<td>20.0</td>
<td>200</td>
</tr>
</tbody>
</table>
A plot of the concentration of B versus time yields a straight line on arithmetic paper indicating a zero order production reaction. The slope of the line is equal to \( k = 5 \) mg/L-h. Species B is being produced. See plot below.

6. If the half-life of a chemical compound is 30 days under anaerobic conditions, determine the first-order removal rate constant, \( k \).

\[
C_t = C_0 e^{-kt} \quad C_t = 0.5 C_0 \quad \text{when} \quad t = t_{1/2}
\]

\[
0.5 C_0 = C_0 e^{-k \cdot 30 \text{d}}
\]

\[
k = 0.023 \text{ h}^{-1}
\]

7. Three wastewater streams are combined at a food processing facility to equalize the pH prior to biological treatment. The flow rate and pH of each of the wastewater streams is presented in the following table. Perform a mass balance on flow and the hydrogen ion concentration (H+) so that the pH of the three combined streams may be estimated. The pH of a solution is equal to the negative logarithm of the hydrogen ion concentration (\( \text{pH} = -\log [\text{H}^+] \)).

<table>
<thead>
<tr>
<th>Wastewater Stream</th>
<th>Flow (liters per minute)</th>
<th>pH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>5.5</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>6.5</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Recall \( \text{pH} = -\log[\text{H}^+] \) See Example 6.8.
\[
\left[H^+\right]_{\text{average}} = \frac{\left[H^+_1\right]Q_1 + \left[H^+_2\right]Q_2 + \left[H^+_3\right]Q_3}{Q_1 + Q_2 + Q_3}
\]

\[
\left[H^+\right]_{\text{average}} = \frac{10^{-5.5} (5 \text{Lpm}) + 10^{-6.5} (20 \text{Lpm}) + 10^{-8.5} (25 \text{Lpm})}{5 + 20 + 25 \text{ Lpm}} = 4.44 \times 10^{-7} \text{ moles/L}
\]

\[\text{pH} = - \log[H^+] = - \log[4.44 \times 10^{-7}] = 6.35\]

8. A sanitary landfill receives 600 ft\(^3\) of municipal solid waste 5 days per week at a density of 500 lb/\(\text{yd}^3\). If the solid waste is compacted to 1000 lb/\(\text{yd}^3\) and the average depth of each cell is 10 feet, estimate the expected life of the landfill in years if 25 acres of space are still available. Draw a materials balance diagram to solve the problem. (1 acre = 43,560 ft\(^2\))

\[\text{SW input} = \left(\frac{600 \text{ ft}^3}{\text{d}}\right) \left(\frac{5 \text{ d}}{\text{wk}}\right) \left(\frac{52 \text{ wk}}{\text{yr}}\right) \left(\frac{1 \text{ yd}^3}{27 \text{ ft}^3}\right) \left(\frac{500 \text{ lb}}{\text{yd}^3}\right) = 2.89 \times 10^6 \text{ lb/yr}\]

\[\rho = 1000 \text{ lb/\(\text{yd}^3\)}\]

\[\text{SW input} = 2.89 \times 10^6 \frac{\text{lb}}{\text{yr}} \left(\frac{1 \text{ yd}^3}{1000 \text{ lb}}\right) = 2889 \frac{\text{yd}^3}{\text{yr}}\]

\[\text{Area} = 25 \text{ ac} \left(\frac{43,560 \text{ ft}^2}{\text{ac}}\right) = 1.089 \times 10^6 \text{ ft}^2\]

\[\text{SW input} = 2889 \frac{\text{yd}^3}{\text{yr}} \left(\frac{27 \text{ ft}^3}{\text{yd}^3}\right) = 7.8 \times 10^4 \frac{\text{ft}^3}{\text{yr}}\]

\[\text{Volume of one 10 ft cell} = (1.089 \times 10^6 \text{ ft}^2)(10 \text{ ft}) = 1.089 \times 10^7 \text{ ft}^3\]

\[\text{Expected life} = \frac{1.089 \times 10^7 \text{ ft}^3}{7.8 \times 10^4 \text{ ft}^3/\text{yr}} = 140 \text{ yr}\]
9. Perform a materials balance on substrate (S) around a chemostat (completely mixed reactor without recycle) assuming a first-order removal (dS/dt = -kS) for substrate with a rate coefficient k value of 0.5 days$^{-1}$. The influent substrate concentration is 150 mg/L and 90% removal is desired. Determine the detention time in hours for the chemostat assuming steady-state conditions.

$$S = (1 - 0.90)150 \frac{mg}{L} = 15 \frac{mg}{L}$$

\[
\left(\frac{dS}{dt}\right)_{\text{accumulation}} V = QS_o - QS + rV \\
\]

$$r = -kS$$

\[
\left(\frac{dS}{dt}\right)_{\text{accumulation}} V = QS_o - QS + -kSV \\
\]

At steady state, the accumulation term goes to zero; and the above equation reduces to the following equation:

$$Q(S_o - S) = kSV$$

$$\theta = \frac{V}{Q} = \frac{(S_o - S)}{kS} = \frac{(150 - 150)mg/L}{0.5h^{-1}(15mg/L)} = 18h$$

10. Calculate the volume of an ideal plug flow reactor for the following scenario.

The volumetric flow rate is 6,500 m$^3$/day and Species A is being removed or converted according to a first-order reaction as follows: dC$_A$/dt = - k C$_A$, where C$_A$ is the concentration of Species A and k = 9000 days$^{-1}$. A 95% removal or conversion of Species A is required.

$$C_t = (1 - 0.95)C_o = 0.05C_o$$

From Table 6.4, use the following equation:
\[ \theta = \frac{\ln(C_o) - \ln(C_i)}{k} = \frac{\ln\left(\frac{C_o}{0.05 C_o}\right)}{9,000 \text{ d}^{-1}} = 3.33 \times 10^{-4} \text{ days} \]

\[ V = \theta Q = 3.33 \times 10^{-4} \text{ days} \left(\frac{6500 \text{ m}^3}{\text{d}}\right) = 2.16 \text{ m}^3 \text{ or } 2164 \text{ L} \]

11. A 1000 MW coal-burning power plant is burning West Virginia bituminous coal with 8% ash content. The power plant is 33% efficient with 35% of the ash settling out in the firing chamber as bottom ash. A simplified schematic diagram is shown below. Assume 3.5 kwh per pound of coal.

a. Determine the rate of coal input to the furnace in kg/day

\[
\text{Input Power} = \frac{\text{Output Power}}{\text{Efficiency}}
\]

\[
\text{Energy}_{\text{IN}} = \frac{\text{Energy}_{\text{OUT}}}{\text{Efficiency}} = \frac{1000 \text{ MW}}{0.33} = 3030 \text{ MW}
\]

\[
\text{Energy}_{\text{Heat}} = 3030 - 1000 = 2030 \text{ MW}
\]
Coal Input = \(3030 \text{ MW} \times \left(\frac{1000 \text{ kW}}{1 \text{ MW}}\right) \times \left(\frac{24 \text{ h}}{\text{d}}\right) \times \left(\frac{1 \text{ lb}}{3.5 \text{ kWh}}\right) \times \left(\frac{454 \text{ g}}{\text{lb}}\right) \times \left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)\)

\[\text{Coal Input = } 9.43 \times 10^6 \text{ kg coal/day}\]

a. Assuming that the electrostatic precipitator (ESP) is 99% efficient, calculate the rate of fly ash emitted to the atmosphere in kg/day.

\[\text{Ash Produced} = 9.43 \times 10^6 \text{ kg} \times \left(\frac{0.08 \text{ kg ash}}{\text{kg coal}}\right) = 7.55 \times 10^5 \frac{\text{kg}}{\text{d}}\]

\[\text{Fly Ash Released to Atmosphere} = (1 - 0.35)(1 - 0.99) \times 7.55 \times 10^5 \frac{\text{kg}}{\text{d}} = 4.91 \times 10^3 \text{ kg/d}\]

**Energy Balance**

b. Draw an energy diagram for the facility and calculate the rate of heat emitted to the environment in kJ/s;

\[\text{Energy}_{\text{IN}} = 3030 \text{ MW} \times \frac{1000 \text{ kW}}{1 \text{ MW}} \times \frac{1 \text{ kJ/s}}{1 \text{ kW}} = 3.30 \times 10^6 \frac{\text{kJ}}{\text{s}}\]

\[\text{Energy}_{\text{OUT(Heat)}} = 2030 \text{ MW} \times \frac{1000 \text{ kW}}{1 \text{ MW}} \times \frac{1 \text{ kJ/s}}{1 \text{ kW}} = 2.03 \times 10^6 \frac{\text{kJ}}{\text{s}}\]

\[\text{Energy}_{\text{OUT(Produced)}} = 1000 \text{ MW} \times \frac{1000 \text{ kW}}{1 \text{ MW}} \times \frac{1 \text{ kJ/s}}{1 \text{ kW}} = 1.10 \times 10^6 \frac{\text{kJ}}{\text{s}}\]
12. A pristine stream flowing at 100 cubic feet per second (cfs) in the Rocky Mountains contains 5 mg/L of suspended solids (SS). During the spring, ice melt conveys 250 mg/L of SS at a rate of 20 cfs into the stream. Determine the concentration of SS in the stream during the spring.

\[
SS_{\text{average}} = \frac{SS_1 Q_1 + SS_2 Q_2}{Q_1 + Q_2}
\]

\[
SS_{\text{average}} = \frac{(5 \text{ mg/L})(100 \text{ cfs}) + (250 \text{ mg/L})(20 \text{ cfs})}{100 \text{ cfs} + 20 \text{ cfs}} = 46 \text{ mg/L}
\]

13. Calculate the minimum rate at which 15°C make-up water from a river must be pumped to evaporative cooling towers for a 1000 MW nuclear power plant. The efficiency of the plant is 32% and all of the waste heat is assumed to be dissipated through evaporative cooling with no direct heat lost to the atmosphere.

\[
\text{Energy}_{\text{in}} = \frac{\text{Energy}_{\text{OUT}}}{\text{Efficiency}} = \frac{1000 \text{ MW}}{0.32} = 3125 \text{ MW}
\]

\[
\text{Energy}_{\text{Heat}} = 3125 - 1000 = 2125 \text{ MW}
\]

Using Equation (6.61) calculate the mass of water required.

\[
\left[\text{Rate of Change in Stored Energy}\right] = \dot{m} c \Delta T
\]

\[
2125 \text{ MW} = \dot{m} \times 4184 \frac{\text{J}}{\text{kg} \cdot ^\circ C} \times 15^\circ C \times \frac{1 \text{ MW}}{10^6 \text{ J/s}}
\]

\[
\dot{m} = 3.38 \times 10^4 \text{ kg/s}
\]

\[
\text{Water Flow} = 3.38 \times 10^4 \frac{\text{kg}}{\text{s}} \times \left( \frac{1 \text{ m}^3}{1000 \text{ kg}} \right) = 3.9 \text{ m}^3/\text{s}
\]